

Natural Kobayashi-Maskawa Model of CP Violation and Flavor Physics

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We explore the possible tie between the naturalness of having a very small strong CP θ parameter in the Kobayashi-Maskawa Model and the flavor symmetry. We provide many examples in which the flavor symmetry group at high energy can naturally give rise to Kobayashi-Maskawa Model at low energy with a naturally small θ .

Introduction

Besides some recent anomalies at the two B factories, the Standard Kobayashi-Maskawa (KM) Model of CP Violation has been amazingly successful. Flavor physics is at the center of the Kobayashi-Maskawa(KM) model, the standard model of CP violation. The origin of flavor mixing is a mismatch between the mass eigenstates and the weak gauge eigenstates. Therefore, within the Standard model, the origin of flavor is the Yukawa couplings which give rise to all fermion masses. In KM model, the CP violation is mediated by the charged current processes that interfere with each other. With three generations, the charge current mixing matrix, the CKM matrix, has exactly one physical phase. Indeed it was first observed by Kobayashi and Maskawa(KM)[1] that the two generation Standard Model cannot support any CP violating phase. (Actually Ref.[1] proposed many different additions to the Standard Model in order to incorporate CP violation, including the KM model and Higgs models). The fact that all three generations have to be involved to create a CP violating phenomena, makes KM model extremely subtle and beautiful model for CP violation. It also makes CP violation tightly connected with flavor physics. This partly explains why the CP violation is small in the kaon system. It is because, at the energy scale of kaon, the heavy third generation almost decouples and, as a result, the effective CP violation is more superweak-like despite the fact the phase in the CKM matrix may not be small at all in some convention (In particular, in Wolfenstein[2] or in Chau-Keung[3] conventions). This difference in the quark masses is of course related to the hierarchy in the eigenvalues of the Yukawa coupling matrices which are bare parameters in Standard Model.

The situation of course changes in the B meson system. The higher masses of B (and B_s) mesons open up many decay channels of different flavor property and make it a great laboratory to study flavor physics and the associated CP violating effect. As a result, there has been a large amount of literature studying the effect of CP viola-

tion in different B decay channels and we shall not dwell into this. It suffices to say that KM model predicts special pattern of CP violating effects in various B (and B_s) decay channels which can be contrasted with the patterns arisen from other CP violating mechanisms in the near future. It is also important to note that any theory that attempts to explain the origin of fermion mass hierarchy will automatically alter the picture of CP violation puts forth in KM model for better or for worse.

There are two often quoted weak points in the KM mechanism. First, it has been shown that KM model alone is insufficient to produce large enough baryon asymmetry in the cosmic evolution. The main difficulties are two folds. First of all, given the current limit of the Higgs mass, the electroweak phase transition tends to be not strongly first order enough to provide the off-equilibrium condition necessary to generate baryon asymmetry. Secondly, the CP violating source in CKM matrix tends to be too small to provide large effects. There are various attempts to extend KM model to overcome the above difficulties and to generate the baryon asymmetry at electroweak scale. Two leading proposals are (1) the minimal supersymmetric extension of Standard Model (MSSM), or (2) two Higgs doublet extension. Of course, the alternative is to give up electroweak baryogenesis and produce baryon or lepton asymmetry at higher energy scale.

The second weakness of KM mechanism is that it does not explain why strong CP θ is so small. There are two levels to this problem, the tree and the loop levels. Since the CP violating phase in KM model is a part of the Yukawa couplings, which are dimension four couplings just like the tree level θ parameter, the θ should naturally be one of the bare parameters of the model with a natural value of order one. The phase convention independent, physical θ , which is a linear combination of three level θ_0 and extra contribution from the quark mass matrices, $\theta = \theta_0 - \text{Arg}(\det M^u M^d)$, should also be naturally of order one. This is the tree level strong CP problem. The second issue is, even if the θ is tuned to zero at tree level, radiative corrections still make it too large or divergent.

However, it should be noted that, by being tightly connected with the flavor physics as described above, the KM mechanism has already embedded in itself a natural mechanism to suppress the loop correction to θ . It has been shown that if one heuristically set the physical θ to zero at the tree level, nonzero loop correction will not occur till three loop level (with two weak and one strong loops)[4], and logarithmic divergent correction to θ can appear only at the 14th order of the electroweak coupling g_2 (or, at the 7-loop level)[5, 6]. Even if one puts in the Planck scale as the estimate of the cut-off, the divergent correction produces only a minute value for θ just like the 3-loop finite corrections. That is, the special $\theta = 0$ point, while not natural, is actually quite robust under radiative corrections. This nice property is a direct consequence of the coupling between flavor physics and CP violation in the KM model. In this sense, the strong CP problem in KM model is more of a tree level problem. All we need is to look for a mechanism beyond Standard model to suppress the tree level θ parameter. In fact, this mechanism does not have to be a low energy mechanism. It can be some features embedded in a high energy theory such as GUT or string theory. A popular class of model is the Nelson-Barr mechanism [7, 8] in which a (softly broken or gauged) flavor symmetry and spontaneously broken CP symmetry are used at high energy in a GUT-like theory to suppress the tree level θ . The phase of the KM model is generated by introducing additional heavy vectorial fermions which can have CP violating mixing with the ordinary fermions.

In models such as those of Nelson-Barr type, one typically has an one loop induced θ at the higher energy scale. This is of course because the corresponding tight connection between CP violation and flavor physics is lost in the extension. (An exception can be found in Ref.[9] in which this tight connection is almost maintained). Such contributions are typically not suppressed by the heavy scale and it is up to the adjustment of the model parameters to make such contributions small enough. While flavor symmetry was used in the examples provided by Ref.[7, 8], it can potentially be replaced by some other symmetry and therefore does not play an essential role. In addition, in this class of models, the strong CP θ parameter receives one loop contribution at high energy and still needs some fine-tuning to make it small. Another recent example[10] involving even more direct use of flavor symmetry, uses the abelian flavor and CP symmetry to make the up (down) quark mass matrix lower (upper) triangular with real diagonals. This will guarantee that tree level θ is zero while still have enough parameter to create KM phase of any magnitude one wishes. No extra fermions are needed in this type of models.

There are other classes of models that are typified in Refs.[11, 12, 13, 14]. In particular, Ref.[11] uses an $SU(3)$ flavor symmetry which gives rise to the one-loop suppression of θ . In Ref.[12], a flavor sensitive discrete symmetry

is used to reduce the contribution to θ to the two-loop level. In Ref.[13], left-right gauge symmetry is employed to reduce the contribution to θ to the two-loop level.

In this paper we investigate a few models that can produce KM model as a low energy effective theory but with naturally small θ . The choice of models to review here is of course a result of our own temporary preference. We will start with a review of the models of Glashow, of Masiero-Yanagita with emphasis on the flavor aspect of the model. Then, we shall present a new model based on an $SO(3)$ horizontal symmetry. We will show that in both $SO(3)$ and $SU(3)$ models, the θ can be reduced to two-loop level by using an additional discrete symmetry which can be broken softly or spontaneously. A more detailed exposition of this new class of models is contained in Ref.[14].

Glashow's Model with Triangular Quark Mass Matrices

Glashow invented a softly broken $U(1)$ flavor symmetry to produce a triangular mass matrix for quarks at the tree level using three Higgs doublets and soft CP breaking. The three Higgs doublets, $H_{(0)}$, $H_{(1)}$ and $H_{(2)}$, couple to the quarks in a way preserving an abelian flavor quantum number F given by

$$F(H_{(k)}) = k, \quad F(u_R^{(i)}) = F(d_R^{(i)}) = F(q_L^{(i)}) = f(i),$$

$$f(i) \equiv \begin{cases} +1, & \text{for } i = 1 \\ 0, & i = 2 \\ -1, & i = 3 \end{cases}.$$

Here the quark field is indexed by its generation.

The symmetry turns the Yukawa Hamiltonian into the form:

$$y_d^{(i,j)} \bar{q}_L^{(i)} H_{(j-i)} d_R^{(j)} + y_u^{(i,j)} \bar{q}_L^{(i)} (i\tau_2)(H_{(i-j)})^* u_R^{(j)}$$

There are no Higgs field with $F = -1$ or $F = -2$, so the Yukawa couplings are of the form:

$$\begin{pmatrix} \bar{d}_L & \bar{s}_L & \bar{b}_L \end{pmatrix} \begin{pmatrix} y_d^{(1,1)} H_0 & y_d^{(1,2)} H_1 & y_d^{(1,3)} H_2 \\ 0 & y_d^{(2,2)} H_0 & y_d^{(2,3)} H_1 \\ 0 & 0 & y_d^{(3,3)} H_0 \end{pmatrix} \begin{pmatrix} d_R \\ s_R \\ b_R \end{pmatrix} \\ + (\bar{u}_L \ \bar{c}_L \ \bar{t}_L) \begin{pmatrix} y_u^{(1,1)} H_0^* & 0 & 0 \\ y_u^{(1,2)} H_1^* & y_u^{(2,2)} H_0^* & 0 \\ y_u^{(1,3)} H_2^* & y_u^{(2,3)} H_1^* & y_u^{(3,3)} H_0^* \end{pmatrix} \begin{pmatrix} u_R \\ c_R \\ t_R \end{pmatrix}.$$

The CP symmetry is assumed to be only softly broken such that the Yukawa couplings as well as the tree level θ coupling are both zero. The vev's of H 's produce the mass matrices for the down-type quarks and the up-type quarks in the form of

$$(\bar{d}_L^{(i)})(M_D)_{ij}(d_R)^{(j)} , \quad (\bar{u}_L^{(i)})(M_U)_{ij}(u_R)^{(j)} ,$$

$$(M_D) = \begin{pmatrix} y_d^{(1,1)}\langle H_0 \rangle & y_d^{(1,2)}\langle H_1 \rangle & y_d^{(1,3)}\langle H_2 \rangle \\ 0 & y_d^{(2,2)}\langle H_0 \rangle & y_d^{(2,3)}\langle H_1 \rangle \\ 0 & 0 & y_d^{(3,3)}\langle H_0 \rangle \end{pmatrix}$$

$$\equiv \begin{pmatrix} m_d^{(0)} & \epsilon_{12} & \epsilon_{13} \\ 0 & m_s^{(0)} & \epsilon_{23} \\ 0 & 0 & m_b^{(0)} \end{pmatrix} ,$$

$$(M_U^{(0)}) = \begin{pmatrix} y_u^{(1,1)}\langle H_0^* \rangle & 0 & 0 \\ y_u^{(1,2)}\langle H_1^* \rangle & y_u^{(2,2)}\langle H_0^* \rangle & 0 \\ y_u^{(1,3)}\langle H_2^* \rangle & y_u^{(2,3)}\langle H_1^* \rangle & y_u^{(3,3)}\langle H_0^* \rangle \end{pmatrix}$$

$$\equiv \begin{pmatrix} m_u^{(0)} & 0 & 0 \\ \epsilon_{21}^* & m_c^{(0)} & 0 \\ \epsilon_{31}^* & \epsilon_{32}^* & m_t^{(0)} \end{pmatrix} ,$$

where $m_i^{(0)}$ are the zeroth order quark mass without the mixing effects. Among the three vev's of the Higgs bosons, $\langle H_0 \rangle$ is chosen to be real by convention. ϵ_{12} , ϵ_{23} , ϵ_{21} and ϵ_{32} all have the same phase and, ϵ_{13} and ϵ_{31} have another. Since both flavor and CP are assumed to be softly broken, one can use the flavor symmetry to remove another phase from the vev's. For example, one can make the phase of ϵ_{12} and ϵ_{23} vanish and be left with complex phase only in ϵ_{13} and ϵ_{31} . Note that one special case is that the phase of ϵ_{13} is twice that of ϵ_{23} . In that case, using the flavor symmetry one can make all vev's real and CP violation disappears from the mass matrices (even though there are still soft CP violation in the Higgs mixing).

In the above choice of the F quantum number, we made M_U lower triangular and M_D upper triangular. However, if we make another choice,

$$F(u_R^{(i)}) = F(d_R^{(i)}) = F(q_L^{(i)}) = -f(i) ,$$

we can have M_U upper triangular but M_D lower. Both are phenomenologically feasible but we take the first choice for its simplicity. With this choice, one then diagonalizes the mass matrix perturbatively and derives the mixing angles

$$V_{12} = \frac{\epsilon_{12}m_s}{m_s^2 - m_d^2} - \frac{\epsilon_{21}^*m_u}{m_c^2 - m_u^2} \approx \frac{\epsilon_{12}}{m_s} - \frac{\epsilon_{21}^*m_u}{m_c^2}$$

$$V_{23} = \frac{\epsilon_{23}m_b}{m_b^2 - m_s^2} - \frac{\epsilon_{32}^*m_c}{m_c^2 - m_t^2} \approx \frac{\epsilon_{23}}{m_b} - \frac{\epsilon_{32}^*m_c}{m_t^2}$$

$$V_{13} = \frac{\epsilon_{13}m_b}{m_b^2 - m_d^2} - \frac{\epsilon_{31}^*m_u}{m_t^2 - m_u^2} \approx \frac{\epsilon_{13}}{m_b} - \frac{\epsilon_{31}^*m_u}{m_t^2}$$

It is clear that the down-quark mass matrix provides the dominant contribution to the mixing angles, with $\epsilon_{12} \approx 25MeV$, $\epsilon_{13} \approx 13MeV$ and $\epsilon_{23} \approx 150MeV$. We can choose the convention that ϵ_{12} , ϵ_{23} are real, and only ϵ_{13} complex in the way consistent to the Wolfenstein (Chau-Keung) parameterization. In this convention, the complex phase in the (ρ, η) plane is the phase of ϵ_{13} , which can be of order ONE!

One should also note that the flavor Abelian group can be reduced from a continuous phase $e^{iF\phi}$ to discrete phase angles, $e^{i\frac{2}{3}\pi F}$, while achieving the same kind of triangular mass matrices.

The above triangular structures are of course not preserved at the one loop level, and as a result, there will be one loop correction which was estimated[10] to be $\Delta\theta = (1/4\pi)^2(\epsilon_{13}\epsilon_{23}^*\epsilon_{21}^*/v_2^2M_u)K$, where K is an one loop integral factor of order one or smaller. This can be about 10^{-9} or smaller. Now, if one takes the masses of the exotic (beyond SM) scalar bosons, as well as the scale of the dimensionful soft flavor breaking scalar couplings, to be around a high energy scale Λ , the resulting one-loop θ depends on Λ only logarithmically through K . Therefore, while these exotic scalar bosons can be as light as TeV , it is not necessary, as far as solving strong CP problem is concern. One can imagine taking Λ to be a very high scale, these exotic scalar bosons all decouple and one is left effectively, at low energy, with a Standard KM Model with small tree level θ .

It is interesting to note that while the model does not explain the fermion hierarchy problem, it does tie up the fermion hierarchy problem with the strong CP problem in the sense that, the smallest of one-loop θ is related to the smallness of off-diagonal ϵ values.

Hermitian Mass matrix

In the model proposed by Masiero and Yanagida[11], the existing three generations are extended with another three generations. The new three generations are individually $SU_L(2)$ singlets and vector-like; nonetheless, the hypercharges are chosen such that they have same electric charges as the known generations. Therefore they are labelled by

$$U_{Li}, U_{Ri}, D_{Li}, D_{Ri} ,$$

in an analogous fashion with the known quarks,

$$q_{Li} \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, u_{Ri}, d_{Ri} .$$

The only difference between the two sets of fermions is in $SU_L(2)$ assignments. A horizontal flavor symmetry

$SU(3)_\parallel$ transforms every Weyl fermion multiplet above in the fundamental representation, labelled by generation index $i = 1, 2, 3$. There are new neutral (and inert to $SU(2)_L \times U(1)$) Higgs bosons: three $SU(3)_\parallel$ octets ϕ_α^a ($a = 1, \dots, 8$, $\alpha = 1, 2, 3$) and a singlet Φ . The Yukawa couplings in the Hamiltonian is

$$\begin{aligned} & \bar{d}_R(g_{d\alpha}\phi_\alpha^a\lambda^a + g'_d\Phi)D_L \\ & + \bar{u}_R(g_{u\alpha}\phi_\alpha^a\lambda^a + g'_u\Phi)U_L \\ & + h_d\bar{D}_RH^\dagger q_L + h_u\bar{U}_R\tilde{H}^\dagger q_L + \text{H.c.} \end{aligned}$$

with the usual $SU_L(2)$ Higgs doublet H which couples to fermions in a way crossing two kinds of fermions. We denote $\tilde{H} \equiv i\tau_2 H^*$.

An alert reader will tell that a criterion is need to distinguish u_R from U_R , or d_R from D_R . Ref.[11] imposed a discrete symmetry under which u_R, d_R and all ϕ, Φ fields are odd, while U and D are even. The vev's of ϕ 's and Φ and H give rise to the 6×6 mass matrix term

$$(\bar{d}_R \ \bar{D}_R) \begin{pmatrix} 0 & g_d\langle\phi\rangle^a\lambda^a + g'_d\langle\Phi\rangle \\ h_d\langle H^\dagger \rangle & M_D \end{pmatrix} \begin{pmatrix} d_L \\ D_L \end{pmatrix}.$$

Here M_D is the heavy allowed vector mass of the D field. The 6×6 matrix has real determinant when couplings are real, as required by the imposed CP symmetry. Note that the singlet is necessary, otherwise the Hermitian mass matrix will be traceless which is not allowed phenomenologically. Also note that to break CP and to have most general Hermitian mass matrix, one need all components of the octets to develop VEV, this is why it is necessary for have three octets. It is not clear whether this has been proved in the literature or not.

Integrating out the exotic generation D , we have the effective mass matrix for the known generations,

$$(1/M_D)h_d\langle H^\dagger \rangle(g_d\langle\phi^a\rangle\lambda^a + g'_d\langle\Phi\rangle),$$

which is Hermitian with real determinant. Some components of Gellmann matrices λ^a are complex to produce the desired CKM phenomenology of CP violation.

The effective mass can be understood as the amplitude given by the diagram below. It is a kind of see-saw mechanism in the quark sector. Similar formulas occur for the up-type quarks. It is interesting to note that the mass matrix is Hermitian certainly because the $SU(3)$ symmetry plus spontaneous CP violation, the fact that ϕ^α are octet. However even after $SU(3)$ is broken, the hermiticity of the mass matrix is still maintained.

strong CP θ issue

As long as the mass matrix is block-Hermitian, the contribution to θ will vanish. So, to look for contribution to θ , one looks for loop induced operator that may violate this hermiticity, such as

$$(1/M^n)\bar{d}_L d_R H f_n(\phi, \Phi)$$

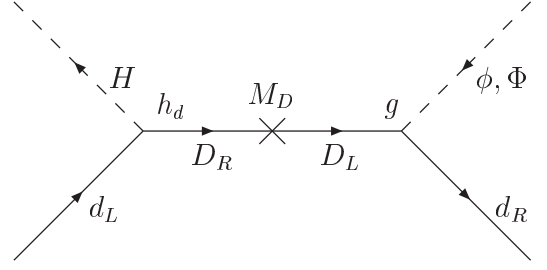


FIG. 1: A see-saw diagram for the effective masses of the known 3 generation.

where f_n is a function of ϕ or Φ . The discrete symmetry requires function f_n to be of even power. However, since the fermion bilinear can only either be $SU(3)$ singlet or octet, f_n has to be effective singlet or octet. Naively it may seem that there will be a one loop contribution to θ at this point. However, as shown in Ref.[14], the one loop θ remains zero in this model.

Note that Higgs scalar potential cannot break parity. It can be done only by chiral fermions. But since P is already broken by the chiral fermion in SM, violation of charge conjugation in the Higgs VEV produces CP violation. Ref.[11] used the nonabelian character of $SU(3)$ to break charge conjugation, C , in the Higgs potential. In principle this can be done by any irrep in nonabelian group as long as one uses enough copies. We call this "nonabelian CP violation". One may wonder what is the simpler nonabelian model that can achieve the same goal. The model in Ref.[14] may be the simplest example one can find.

Minimal model of this Class

In Ref.[14], the flavor symmetry is replaced by smaller $SO(3)$ symmetry. The new three generations are individually $SU_L(2)$ singlets and vector-like, nonetheless, the hypercharges are chosen that they have same electric charges as the known generations. Therefore they are labelled by

$$U_{Li}, U_{Ri}, D_{Li}, D_{Ri},$$

in an analogous fashion with the known quarks,

$$q_{Li} \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, u_{Ri}, d_{Ri}.$$

A horizontal flavor symmetry $SO(3)_\parallel$ transforms every Weyl fermion multiplet above in the $\mathbf{3}$ representation, labelled by generation index $i = 1, 2, 3$. There are new horizontal neutral (inert to $SU(2)_L \times U(1)$) Higgs bosons, *i.e.* one quintet (symmetric traceless rank-2 tensor) CP-even ϕ_S and one triplet (antisymmetric rank-2 tensor)

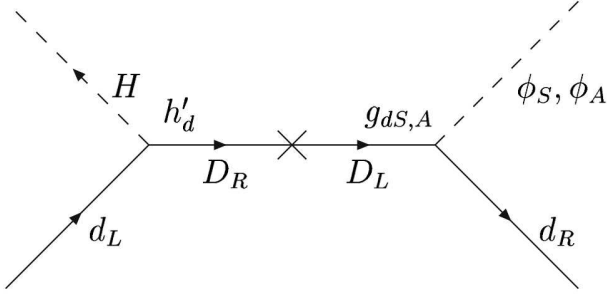


FIG. 2: A see-saw diagram for the effective masses of the known 3 generation.

CP-odd ϕ_A . The Yukawa couplings are

$$\begin{aligned} & \bar{d}_R(\mu_d + g_{dS}\phi_S + ig_{dA}\phi_A)D_L \\ & + \bar{u}_R(\mu_u + g_{uS}\phi_S + ig_{uA}\phi_A)U_L \\ & + \bar{D}_R(\mu_D + g_{DS}\phi_S + ig_{DA}\phi_A)D_L \\ & + \bar{U}_R(\mu_U + g_{US}\phi_S + ig_{UA}\phi_A)U_L \\ & + (h_d\bar{d}_R + h'_d\bar{d}_R)H^\dagger q_L \\ & + (h_u\bar{u}_R + h'_u\bar{u}_R)\tilde{H}^\dagger q_L + \text{H.c.} \end{aligned}$$

with the usual $SU_L(2)$ Higgs doublet H which couples to fermions flavor-blindly. We denote $\tilde{H} \equiv i\tau_2 H^*$.

The vev's of ϕ_S and ϕ_A and H give rise to the following 6×6 mass matrix term

$$(\bar{d}_R \ \bar{D}_R)(M_6) \begin{pmatrix} d_L \\ D_L \end{pmatrix},$$

$$(M_6) = \begin{pmatrix} h_d\langle H^\dagger \rangle \mathbf{1} & \mu_d + g_{dS}\langle \phi_S \rangle + ig_{dA}\langle \phi_A \rangle \\ h'_d\langle H^\dagger \rangle \mathbf{1} & \mu_D + g_{DS}\langle \phi_S \rangle + ig_{DA}\langle \phi_A \rangle \end{pmatrix}$$

The 6×6 matrix has real determinant when couplings are real, as required by the imposed CP symmetry. Contrary to the $SU(3)$ model, it is not necessary to have a $SO(3)$ singlet if one does not impose a discrete symmetry.

Integrating out the exotic generation D , we have the effective mass matrix for the known generations like CKM phenomenology. The effective mass can be understood as the amplitude given by the diagram below. It is a kind of see-saw mechanism in the quark sector. Similar formulas occur for the up-type quarks. The low energy effective mass matrix can be obtained through the following steps. In the limit of $\langle H \rangle = 0$, d_L quarks decouple and we have the reduced mass matrix of a size 6×3 instead,

$$(\bar{d}_R \ \bar{D}_R) \begin{pmatrix} M_d \\ M_D \end{pmatrix} (D_L),$$

$$\begin{aligned} M_d &= \mu_d + g_{dS}\langle \phi_S \rangle + ig_{dA}\langle \phi_A \rangle, \\ M_D &= \mu_D + g_{DS}\langle \phi_S \rangle + ig_{DA}\langle \phi_A \rangle. \end{aligned}$$

We find a 6×6 unitary matrix V to transform the the above 6×3 into one with zero entries in the upper 3×3

block.

$$V \begin{pmatrix} M_d \\ M_D \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ M'_D \end{pmatrix},$$

by choosing the top three row vectors of V perpendicular to the 3 column vectors in the mass matrix. It is possible because the 6 dimensional linear space is larger than the 3 dimensional space spanned by the three column vectors. Furthermore, by using bi-unitary transformation, we also rotate D_L into D'_L so that M'_D is diagonal. In this way, the massless states d'_R and the massive states D'_R are generally mixed among the original d_R and D_R . Nonetheless, the three generations of d_L remain massless and unmixed.

We include the effect $\langle H \rangle$ from the viewpoint of perturbation. The mass terms involving d_L are tabulated in the matrix form,

$$(\bar{d}_R \ \bar{D}'_R)V \begin{pmatrix} h_d\langle H^\dagger \rangle \mathbf{1} \\ h'_d\langle H^\dagger \rangle \mathbf{1} \end{pmatrix} (d_L) = (\bar{d}_R \ \bar{D}'_R) \begin{pmatrix} \hat{m}_d \\ \hat{m}'_d \end{pmatrix} (d_L).$$

Including D'_L , we have

$$(\bar{d}'_R \ \bar{D}'_R) \begin{pmatrix} \hat{m}_d & \mathbf{0} \\ \hat{m}'_d & M'_D \end{pmatrix} \begin{pmatrix} d_L \\ D'_L \end{pmatrix}.$$

As $M'_D \gg \hat{m}_d$, masses of usual d -quarks are basically given by diagonalization of the mass matrix \hat{m}_d . Similar procedures also apply to the u -quarks. Phenomenology of CKM mechanism follows.

The model as it is will have one-loop contribution to θ . To make the one-loop contribution vanishes, it is necessary to make the $\bar{d}_R d_L$ block in the mass matrix M_6 vanishes at tree level. This can be easily done with a discrete symmetry as explained in Ref.[14]. This will make the θ of the low energy effective KM model naturally small. Note that the smallness is not due to the suppressive effect of any high energy scale. It is a consequence of natural smallness of the two loop quantum effect.

Comparison with Nelson-Barr scheme

Let us make a brief comparison with Nelson-Barr[7, 8] scheme before we conclude. They proposed the model in grand unified theory (GUT) context. To compare, we first strip the model off the GUT context. For the extra heavy fermions, they use an additional set of Standard Model fermions plus its parity mirror (to make it a vectorial set). They impose flavor symmetry on the SM fermions, but not the extra fermions. Therefore their heavy fermions are flavor singlets and one needs to use only one copy, that is,

$$Q_L \equiv \begin{pmatrix} U_{Li} \\ D_L \end{pmatrix}, U_R, D_R, Q'_R \equiv \begin{pmatrix} U'_R \\ D'_R \end{pmatrix}, U'_L, D'_L,$$

in addition to the known quarks,

$$q_{Li} \equiv \begin{pmatrix} u_{Li} \\ d_{Li} \end{pmatrix}, u_{Ri}, d_{Ri}.$$

The flavor symmetry is broken by a set of $SU(2)$ -singlet Higgs fields Φ, Φ' that connect the light and heavy fermions. The vev's of Φ and H give rise to the 5×5 mass matrix term

$$(\bar{d}_{Ri} \ \bar{D}'_R \ \bar{D}_R) \begin{pmatrix} \lambda \langle H^\dagger \rangle & 0 & f \langle \Phi \rangle \\ f' \langle \Phi' \rangle & M_{D_1} & 0 \\ 0 & 0 & M_{D_2} \end{pmatrix} \begin{pmatrix} d_{Li} \\ D_L \\ D'_L \end{pmatrix}.$$

Here $M_{D_{1,2}}$ are the heavy Dirac masses of the D and D' fields. CP symmetry is arranged such that it is broken in the off-diagonal $f \langle \Phi \rangle$ and $f' \langle \Phi' \rangle$ components. Note that these components do not contribute to the mass determinant. The 5×5 matrix has real determinant when couplings are real, as required by the imposed CP symmetry. Clearly, one can see the difference between the Nelson-Barr mechanism and the block-Hermitian mechanism.

Conclusion

There are many solutions to the strong CP problem. Axionic models are among the most popular. It is not difficult to include axion in grand unified theories or even string theory. However, it may be interesting to think that strong CP problem may be our best hint to some unknown high energy physics. One of the other current puzzles of particle physics which we also wish to be resolved by some unknown high energy physics is the flavor problem including the mass hierarchy problem. One may wonder if the two puzzles may have the same solution which is the flavor symmetry. In this paper we explore

this possibility to some extent. Certainly there are a lot more possibilities to cover.

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- [1] M. Kobayashi and T. Maskawa, Prog. Theor. Phys. **49** (1973) 652.
 - [2] L. Wolfenstein, Phys. Rev. Lett. **51**, 1945 (1983).
 - [3] L. L. Chau and W. Y. Keung, Phys. Rev. Lett. **53**, 1802 (1984).
 - [4] E. P. Shabalin, Sov. J. Nucl. Phys. **36**, 575 (1982) [Yad. Fiz. **36**, 981 (1982)].
 - [5] J. R. Ellis and M. K. Gaillard, Nucl. Phys. B **150**, 141 (1979).
 - [6] I. B. Khriplovich and A. I. Vainshtein, Nucl. Phys. B **414**, 27 (1994) [arXiv:hep-ph/9308334].
 - [7] A. E. Nelson, Phys. Lett. B **143**, 165 (1984); Phys. Lett. B **136**, 387 (1984).
 - [8] S. M. Barr, Phys. Rev. Lett. **53**, 329 (1984), Phys. Rev. D **30**, 1805 (1984).
 - [9] S. M. Barr, D. Chang and G. Senjanovic Phys. Rev. Lett. **67**, 2765 (1991).
 - [10] S. L. Glashow, arXiv:hep-ph/0110178.
 - [11] A. Masiero and T. Yanagida, arXiv:hep-ph/9812225.
 - [12] L. Lavoura, Phys. Rev. D **61**, 076003 (2000) [arXiv:hep-ph/9909275].
 - [13] K.S. Babu, R. N. Mohapatra Phys. Rev. D **41** 1286 (1990), 076003.
 - [14] D. Chang, W.-Y. Keung, arXiv:hep-ph/0312139
 - [15] L. Bento, G. C. Branco and P. A. Parada, Phys. Lett. B **267**, 95 (1991).
 - [16] D. Bowser-Chao, D. Chang and W. Y. Keung, Phys. Rev. Lett. **81**, 2028 (1998) [arXiv:hep-ph/9803273].